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TURBULENT HEAT AND MOMENTUM TRANSFER FOR GASES IN A CIRCULAR TUBE AT WALL TO BULK TEMPERATURE RATIOS TO SEVEN

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Abstract—Measurements are reported for local friction and heat-transfer coefficients with constant heat flux in an electrically heated circular tube. Precooled nitrogen in turbulent flow was used to obtain local wall to bulk temperature ratios from 1·24 to 7·54. The Inconel test section had a 40 diameter hydrodynamic entry section and a 160 diameter heated section. The bulk inlet Reynolds numbers covered the range from 18 300 to 279 000. Correlations in terms of both the bulk and the modified wall Reynolds numbers are presented for the local friction factors; the local heat-transfer results are presented in terms of the wall to bulk temperature ratio with bulk properties and also directly on the modified wall Reynolds number. Both the friction and heat-transfer results are shown to be significantly affected by the influence of the variable properties. Entrance region effects for both the heat-transfer and friction coefficients are discussed.

NOMENCLATURE

D,	diameter;
G,	mass velocity [lbm/ft^{2}h];
g_c,	unit conversion 32·2 [lbm ft/lbfs^2];
H,	enthalpy;
h,	heat-transfer coefficient [Btu/h ft^2 degF];
J,	unit conversion 778 [ft lbf/Btu];
L,	length of test section;
p,	pressure;
q'',	heat flux [Btu/h ft^2];
R,	gas constant;
T,	temperature;
u,	velocity in x direction;
V,	bulk average velocity;
w,	mass flow rate [lbm/h];
x,	distance down test section;
Z,	compressibility factor;
N_{Re},	Reynolds number, $4w/\pi D \mu$;
N_{Nu},	Nusselt number, hD/k;
N_{Pr},	Prandtl number, $\mu c_p/k$.

Greek symbols

ρ,	density [lbm/ft^3];
ϕ,	impulse function, see equation (5);
τ,	shear stress;
μ,	viscosity.

Subscripts

aw,	adiabatic wall;
iso,	isothermal conditions;
o,	stagnation conditions;
w,	wall;
b,	bulk.

Note: The absence of a subscript on dimensionless parameters and gas properties denotes bulk static properties.

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INTRODUCTION

In the light of present day technological interests, particularly nuclear reactor power generation and rocket propulsion, convective heat-transfer problems are becoming more involved with the effects of variable properties. Because of interest in these effects an experimental program has been under way at Stanford University for several years to measure friction and heat-transfer coefficients with high wall to bulk temperature ratios [1, 2], the wall to bulk

Table 1. Selected variable properties, turbulent heat-transfer experiments

Investigators	Gas	$T_w/T_{b_{max}}$	x/D	Reference temperature	Correlation constant	Exponent on T_w/T_b	Entry parameter	Remarks
	Overall average coefficients							
Humble *et al.* [3]	Air	3·5	30–120	film	0·034	0	$(L/D)^{-0 \cdot 1}$ to 60	—
McCarthy and Wolf [4]	Hydrogen }	9·9	21–67	film	0·045	−0·7	$(L/D)^{-0 \cdot 15}$	$Re_f = 4w/\pi D\mu_f$
	Helium }	9·9	21–67	wall	0·045	0	$(L/D)^{-0 \cdot 15}$	Modified *Re*
		9·9	21–67	bulk	0·045	−0·55	$(L/D)^{-0 \cdot 15}$	—
Taylor and Kirchgessner [19]	Helium	2·7	60, 92	film	0·021	0	$1 + (L/D)^{-0 \cdot 7}$	—
	Incremental average coefficients							
Taylor and Kirchgessner [19]	Helium	3·9	60, 92	film	0·021	0	—	Modified *Re*
				wall	0·0265	0	—	Modified *Re*
Taylor [8]	Hydrogen Helium	5·6	77	film	0·021	0	—	Modified *Re*
McCarthy and Wolf [4]	Hydrogen }	11·1	21–67	wall	0·025	0	0	Modified *Re*
	Helium }			bulk	0·025	−0·55	—	—
	Local coefficients							
Wieland [5]	Helium }	2·83	250	film	0·021	0	Downstream only	Modified *Re*
	Hydrogen }			wall	0·021	—	Downstream only	Modified *Re*
McEligot *et al.* [1]	Air Helium Nitrogen	2·5	160	bulk	0·021	−0·5	$1 + (x/D)^{-0 \cdot 7}$	—
Lel'chuk and Dyadyakin [9]	Air	2·4	147	wall				—
				bulk	0·025	graphic	x/D dependence	
Dalle Donne and Bowditch [6, 7]	Air Helium	2·4	180, 351	wall	0·0208	0	—	Modified *Re*
				bulk	0·022	x/D in exponent on T_w/T_b		—
				entry	0·022	$(T_w/T_{ent.})^{-0 \cdot 30}$		
Present experiment	Nitrogen	7·5		wall	0·023	0	$1 + (x/D)^{-0 \cdot 7} (T_w/T_b)^{0 \cdot 7}$	Modified *Re*
				bulk	0·024	−0·7	$1 + (x/D)^{-0 \cdot 7} (T_w/T_b)^{0 \cdot 7}$	—

temperature ratio being a measure of the effect of the property variation across the flow stream. In order to attain greater ratios than those previously reached, on the order of 2·5, precooled gas was used so that the bulk temperature at the test section inlet was as low as 180°R. Consequently it was possible to reach local wall to bulk temperature ratios as high as 7·54.

The heat-transfer apparatus consisted of an Inconel tube with a 40 diameter hydrodynamic entry section and a 160 diameter heated section. Ideally this results in a thermal boundary condition of uniform inlet temperature and a step change in heat flux at the wall. Local friction coefficients at x/D's from 16 to 114 were determined from pressure drop information taken with 7 pressure taps. Thermocouples were used to measure the local wall temperature variations from which the local heat-transfer coefficients could be determined. Average heat-transfer and friction coefficients were found from the integrated local wall temperatures and the mean properties in the manner of Humble *et al.* [3].

Previous results for turbulent internal flow in ducts with precooled gas include those of McCarthy and Wolf [4] and Wieland [5]. In reference 4 average heat-transfer coefficients were measured with hydrogen and helium gas flowing in electrically heated circular tubes with length to diameter ratios from 20 to 67. Because of the use of these relatively short test sections, a large entry effect on the average heat-transfer results was found. Some local data over the downstream segment of the tube are also given. Correlations were determined on the bulk, film, and wall temperature properties with the use of the wall to bulk temperature ratio to some power. Wieland measured local heat-transfer coefficients with hydrogen gas flowing in an electrically heated circular tube of some 250 diameters in length. He correlated his data with both the wall and film modified Reynolds number. In both references it was found that for large wall to bulk temperature ratios, three or greater, a quite different wall temperature profile resulted than that for the ordinary constant heat flux case. Instead of the usual entrance region rise followed by a linear increase in wall temperature, a sharp rise and then a drop in the wall temperature some 20 to 40 diameters downstream from the entrance was found.

Other work on high-temperature heat transfer in a circular tube includes that of McEligot *et al.* [1], Dalle Donne and Bowditch [6, 7], Taylor [8], and Lel'chuk and Dyadyakin [9]. In these cases precooling was not used and typical values of the local wall to bulk temperature ratio were 2·5 or less. Taylor did achieve ratios up to 5·6 by using a tungsten tube which permitted wall temperatures to 5600°R. Table 1 summarizes the work and heat-transfer results of the various authors. Previously, there have been no local friction coefficient results for temperature ratios greater than 2·5. In the present study, however, local friction coefficients were found at temperature ratios above 7. Table 2 summarizes the range of variables covered in the present work, while Table 3 summarizes the present data.

Table 2. Range of variables in the present experiment

Experimental runs	39
Inlet bulk Reynolds number	18 300–359 000
Exit bulk Reynolds number	4300–204 000
Maximum T_w/T_b	7·54
Axial distances	
Heat-transfer coefficient	1·2–144·0
Friction coefficient	16·0–114·0
Maximum Mach number	
Heat-transfer coefficient	0·50
Friction coefficient	0·46

EXPERIMENTAL APPARATUS

The heat-transfer loop and associated instrumentation have previously been described in references 1 and 10. Only a brief description will be given here. The loop consists of a gas supply bank, followed by a pressure regulator, a system of filters, and an integral-orifice differential pressure flow meter. The gas passes through the liquid-nitrogen cooler to the inlet mixer and then enters the test section after which it is cooled and exhausted to the atmosphere. The flow rate is controlled with a needle valve downstream from the test section. For the cold gas tests a stainless steel liquid-nitrogen dewar was installed with a cooling coil. Figure 1 shows the details of the dewar and test section construction. A second container was used inside the dewar to control the temperature of the lower electrode on the

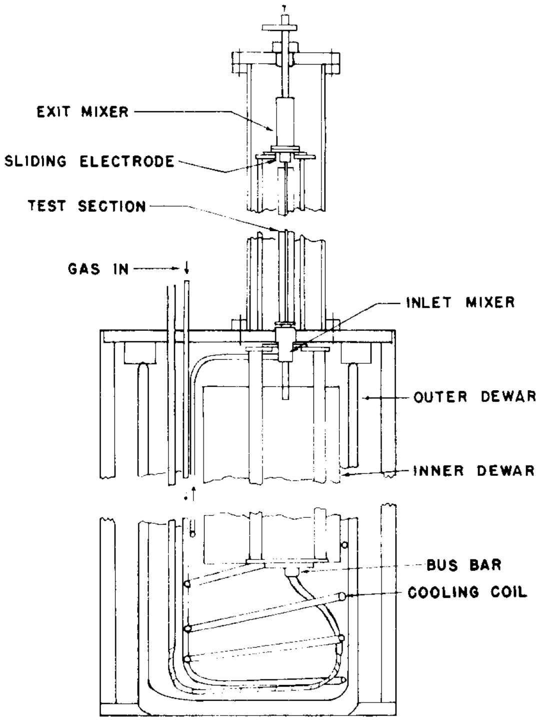

FIG. 1. Test section and cooling dewar.

test section independently of the inlet gas temperature. Thus the temperature difference between the mixer where the inlet temperature was measured and the lower electrode, where the heating began, could be minimized. By regulating the liquid nitrogen level in the two containers sufficient control was available so that little heat transfer to the gas took place in the 40 diameter hydrodynamic development section between the mixer and the lower electrode.

The test section consisted of an Inconel tube of 0·0216 inch wall and 0·1237 in inside diameter, some 26 in long including both the hydrodynamic, unheated, entry length and several diameters length after the last thermocouple. The useful length of the test section was some 144 diameters for heat transfer and 133 diameters for friction factors. Chromel–Alumel thermocouples were spot welded to the test section for temperature measurements and thin wall, 0·006 inch, 0·125 in outside diameter, Inconel pressure taps were brazed to the test section for local pressure measurements. In all, thirteen thermocouples and ten pressure taps were attached to the tube. The pressure taps were drilled out with a 0·015 in drill so the tap to tube diameter ratio was about 0·1. Because of the difficulty in brazing and drilling the taps only seven of the ten taps were usable.

Pressure drops were measured on a gas over water and gas over mercury manometer bank. When the pressure drops were too small for accurate measurements on the vertical manometers an inclined water manometer was used. With this system, pressure drops from 0·1 in water to 60 in of mercury from tap to tap or from any combination of taps could be determined.

Since the tube was heated using a d.c. welding generator and the thermocouples were directly attached some pick-up by the latter was noted. This was corrected for in the manner of Davenport *et al.* [11]. Several tests were conducted with heating but no flow through the tube to determine the local heat loss to the room during the actual test runs. The assumption was made that there was the same heat loss to the room with flow as occurred with no flow at the same wall temperature. This assumes similarity of wall temperature profiles between the heat loss runs and the actual flow runs. With the exception of the first diameters downstream from the start of heating this was the case.

DEFINITIONS AND CALCULATIONS

The local heat-transfer coefficient is defined by the equation

$$q_w'' = h\,[T_w - T_{aw}] \tag{1}$$

The adiabatic wall temperature is determined from the relationship $T_{aw} = T + R\,V^2/2g_c$ where R is the recovery factor for turbulent flow taken to be $N_{Pr}^{1/3}$. The bulk static temperature is defined, by analogy to the energy equation for constant properties treatments, by the equation

$$C_{p,\,o}\,[T_o - T] = V^2/2g_cJ \tag{2}$$

which uses the definition of the bulk velocity as

$$V = GZRT/p \tag{3}$$

A more exact determination would require a knowledge of the actual variation over the cross section of the two-dimensional functions for H, ρ, and u. The local friction factor is defined as

$$f = \tau_w/(\rho V^2/2g_c) \tag{4}$$

Momentum considerations for the steady flow of fluids through circular tubes lead to the following expression for the wall shear stress, τ_w, in terms of quantities which may be determined experimentally:

$$\tau_w = -\frac{D}{4}\frac{\mathrm{d}}{\mathrm{d}x}\left[p + \rho\frac{V^2}{2g_c}\right] = -\frac{D}{4}\frac{\mathrm{d}\phi}{\mathrm{d}x}. \tag{5}$$

This expression, which neglects the change in elevation, assumes that the static pressure is uniform across the flow cross section and treats the momentum flux as one-dimensional.

The data reduction was performed on the IBM 7090 at the Stanford University Computation Center. The input consists of the thermocouple readings converted to °F, flow rate information, pressure drops from tap to tap, etc. The thermocouple data was converted using *NBS 561*, Reference Tables for Thermocouples, which gives Chromel–Alumel e.m.f.'s to −310°F. Formulae and tables contained in the program provide gas properties which were taken from *NBS 564* [12]. Test section characteristics and overall coefficients for the radial heat loss are introduced when needed. In preliminary calculations the program computes the gas flow rate and corrects for the thermal expansion at the thermocouple and pressure tap locations.

Heat-transfer results are obtained at the wall thermocouple locations. The wall heat flux is obtained by subtracting the radial and axial heat losses from the total electrical energy generation. The inside wall temperature is determined from the heat flux and the outside wall temperature. Trapezoidal integration of q''_w then yields the bulk stagnation enthalpy, hence the bulk stagnation temperature. After the bulk static temperature is found, the desired heat-transfer parameters can be calculated.

Local friction factors are determined at the pressure tap locations by interpolating the bulk static temperature to form the specific impulse function, ϕ, then taking its gradient to obtain the wall shear stress. Since the diameter enters these calculations to approximately the fifth power it is corrected for thermal expansion.

Average heat-transfer and friction results are calculated in the manner of Humble *et al.* [3]. Because the present results have inlet temperatures to 180°R it was necessary to extrapolate the *NBS* properties for nitrogen by some 10°R.

RESULTS AND CORRELATIONS

The correlation of heat-transfer and friction data with high heating rates is complicated by the appreciable property variation across the tube. There are two ways which are commonly used to account for the influence of the property variation: one is to use a reference temperature, selected in such a way that when the properties are evaluated at this temperature the correlation reduces to that for constant properties; the second is to correct for the property effect by using a property or temperature ratio in the correlation which otherwise uses bulk properties. Both methods were used to correlate the present data.

Because of the very large property influence in the present experiment the bulk Reynolds number could decrease by as much as a factor of 5 between the tube inlet and exit. In contrast the wall Reynolds number, defined as:

$$N_{Re,\,w} = \frac{4w}{\pi D \mu_w}\left[\frac{T_b}{T_w}\right]$$

first decreased rapidly and then gradually increased along the tube, decreasing by as much as a factor of five but then returning at the exit to within a factor of two of the inlet wall Reynolds number. The film Reynolds number is defined in a similar manner using the viscosity evaluated at a temperature mid-way between the wall and the bulk temperature. It followed the same trend as the wall Reynolds number. When used in this manner the wall and film Reynolds number are commonly referred to as modified Reynolds numbers. For a perfect gas we see that the wall Reynolds number becomes equivalent to

$$N_{Re,\,w} = \left[\frac{V_b\,\rho_b\,D}{\mu_w}\right]\frac{\rho_w}{\rho_b} = \frac{V_b\,\rho_w\,D}{\mu_w}$$

In a paper dealing with ways of correlating heat-transfer data for turbulent gas flow in

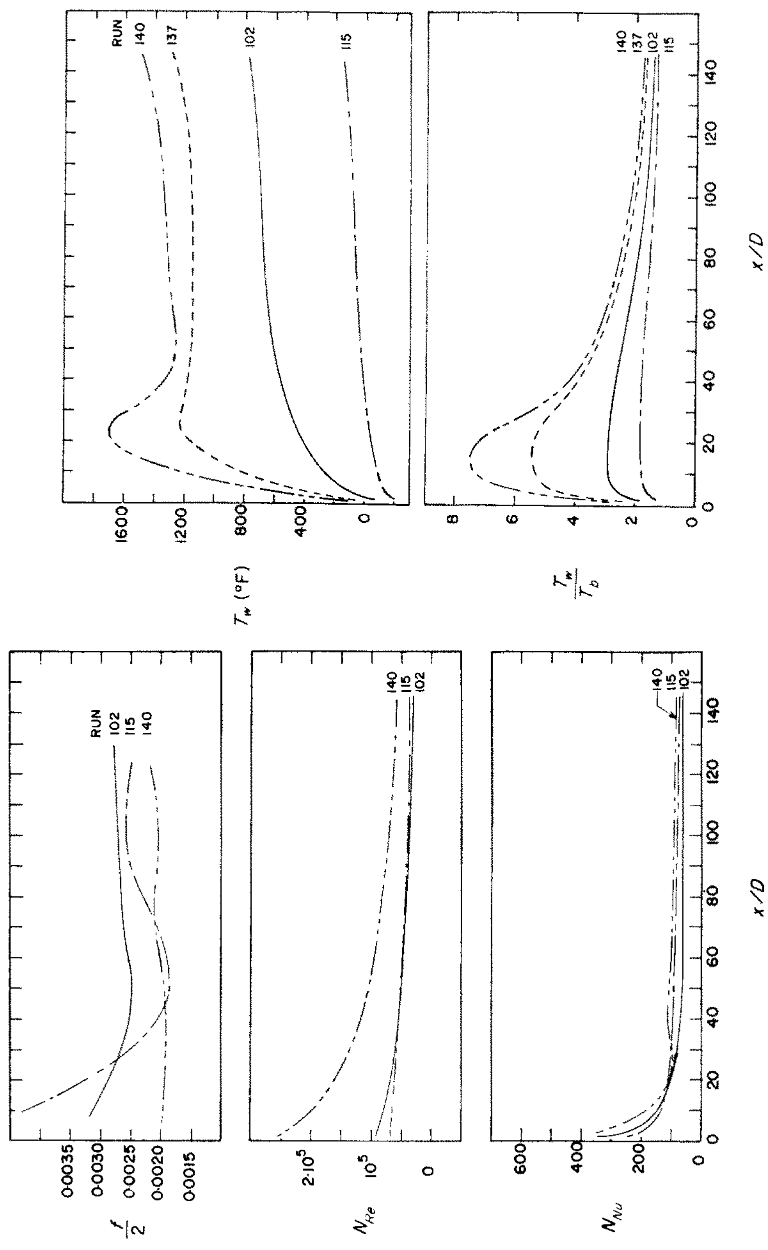

FIG. 2. Variation of experimental parameters with x/D.

tubes with appreciable variation of properties, Smith [13] strongly advocates the use of bulk properties with the use of a wall to bulk temperature ratio to account for the property variation. His arguments are mainly based on the fact that an overall energy balance is directly related to the bulk Stanton number. It may, however, be argued with equal strength that since both heat transfer and wall shear stress in turbulent flow are primarily influenced by the flow field near the wall, either the film or wall properties should be used. The latter point of view gains some support from the fact that in the present study transition from turbulent to laminar-like flow occurred at Reynolds numbers, based on bulk properties, far above the critical value.

Since neither of the two methods can fully take into account the rather complex consequences of the property variation, the choice between them is largely a matter of convenience, particularly convenience in application. From this point of view the use of bulk properties may have an advantage, at least for heat-transfer calculations.

From equation (5) one can note that the friction coefficient is determined from the results of a momentum balance which includes the wall shear stress and the bulk momentum flux. One might thus be tempted to correlate the *bulk* friction factors with the *wall* Reynolds number in the manner of Dalle Donne and Bowditch [7].

With these comments in mind we now turn to the results of the present experiment. Figure 2 shows the typical trends for runs with low, moderate and high wall to bulk temperature ratios. Although not shown, the bulk temperature rises linearly as expected for the case of a constant heat flux and the heat flux is nearly constant after the first few diameters which are affected by the axial heat loss. The temperature ratio is seen to rise to a maximum and then to decay towards the value of 1, while the bulk

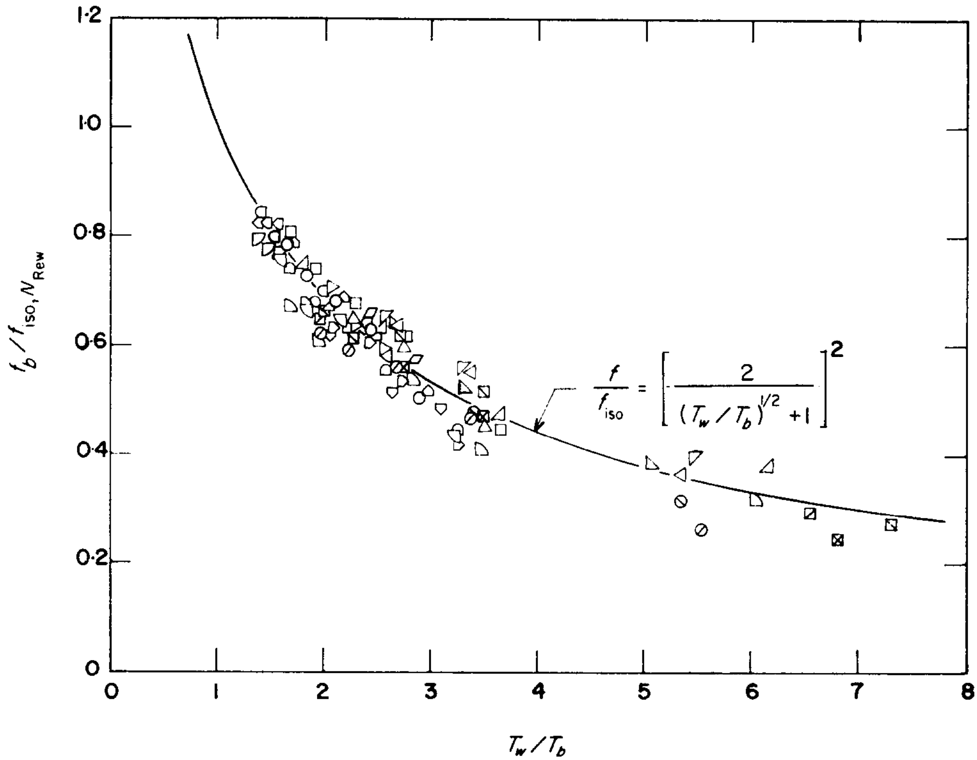

FIG. 3. Local friction coefficients normalized on isothermal wall conditions, $16 \leqslant x/D \leqslant 114$. Correlation as suggested by Kutateladze and Leont'ev [14].

Reynolds number decreases all along the length of the tube. In contrast the wall temperature profile is greatly affected by the degree of property variation present; that is, by the wall to bulk temperature ratio. With large heat fluxes the wall temperature rises as expected but reaches a peak, then decreases, then begins the linear rise characteristic of a constant property, constant heat flux case. This shape profile can be predicted from an equation of the Dittus–Boelter form at large wall to bulk temperature ratios so its existence does not rule out that an equation of that form should provide an adequate correlation of the data.

Friction results

Several runs were made to determine friction coefficients without heating for comparison to the usual correlations for adiabatic friction coefficients. Unfortunately there seems to have been a burr on the test section between taps 2 and 3 causing the isothermal data at tap 2 to fall some 30 per cent higher than the Karman–Nikuradse line while that at tap 3 lay some 7 per cent below the line. Taps 4, 5, and 6 showed good agreement with the Karman–Nikuradse correlation, all points being within 10 per cent of the line. Since the computer program for data reduction fits a curve to the pressure distributions for each group of three pressure taps, friction coefficients are not determined at the first and last taps, hence only at taps 2 through 6 are local friction coefficients determined. Because the correlations based on the present data are given in terms of f/f_{iso} the effect of the burr is eliminated and the data are generally applicable.

Figure 3 shows the result of a correlation of the friction coefficients based on bulk properties but normalized by the (experimental) isothermal friction coefficients for the same modified wall Reynolds number. The solid curve represents the relation

$$f/f_{\text{iso}} = \left[\frac{2}{(T_w/T_b)^{1/2} + 1}\right]^2 \tag{6}$$

obtained by Kutateladze and Leont'ev [14] from an analysis of the non-isothermal boundary

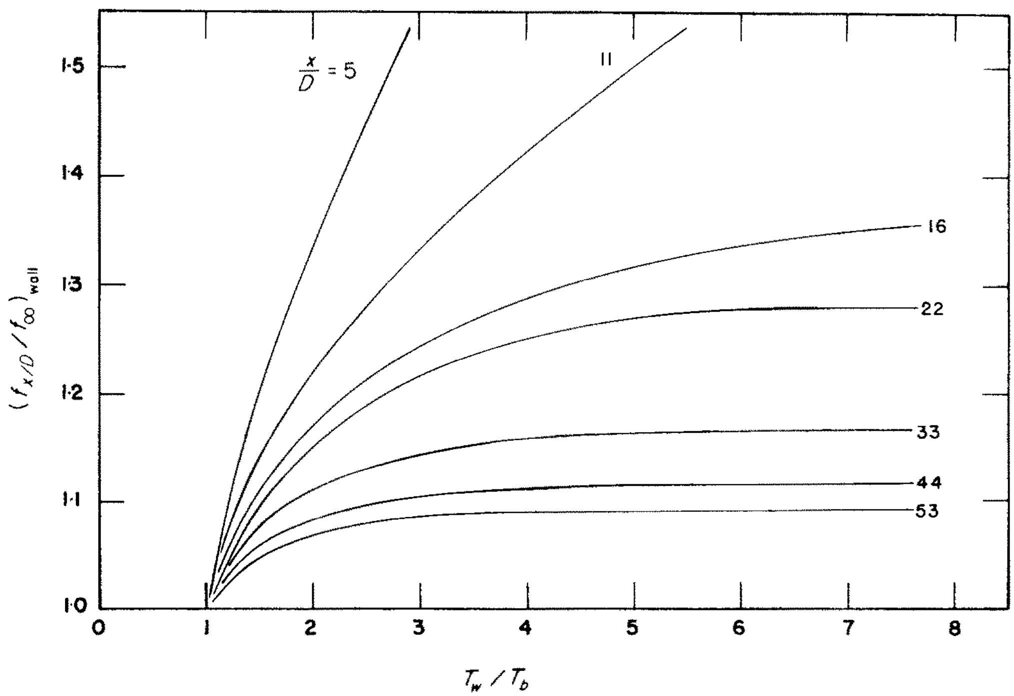

FIG. 4. Entry corrections for friction coefficients based on wall properties.

layer on a flat plate. The expression, which in the above form is valid for the limiting case of subsonic flow at large Reynolds number, has been applied by the above authors to fully developed flow in a tube, however, with the isothermal friction factor based on the bulk Reynolds number. A more exact expression for the finite Reynolds numbers includes an implicit dependence on the isothermal friction factor as well as the ratio of the centerline to bulk velocities. It is somewhat surprising that with wall Reynolds numbers as low as those found here, 5000–50 000, the agreement with the limiting form for large Reynolds numbers is so good. The data at taps 2 and 3 which were located 16 and 53 diameters downstream from the start of heating are corrected for the effect of the thermal entrance region. The corrections were determined in the following manner:

1. The ratio of $f_b/f_{\text{iso},\,w}$ was plotted versus the temperature ratio, T_w/T_b, for each of the five taps at which local friction coefficients were determined.
2. A cross plot was then made of $f_b/f_{\text{iso},\,w}$ versus x/D for a constant value of the temperature ratio.
3. From this the ratio of $f_b/f_{\text{iso},\,w}$ divided by $f_b/f_{\text{iso},\,w}$ downstream, was determined as a function of x/D in the entrance region.

The entry correction, shown in Fig. 4, thus involved finding the parameter necessary to reduce the upstream data to the downstream values. Since the upstream results were taken at wall to bulk temperature ratios to 7·47 and the downstream data only covered ratios as high as 3·5 it was necessary to extrapolate the downstream results to these higher temperature ratios. However, the results from the upstream pressure taps paralleled those of the downstream taps so there is little uncertainty in doing this.

As a somewhat simpler alternative to the formulation suggested by Kutateladze and Leont'ev, $f_b(f_{KN}/f_{\text{iso}})_{N_{Re,\,w}}$ was plotted versus the modified wall Reynolds number. For taps 4, 5 and 6 this was equivalent to using f_b while for taps 2 and 3 this method eliminates the effect of the burr between taps 2 and 3. It is seen in Fig. 5(a) that the factor $(f_{KN}/f_{\text{iso}})_{N_{Re,\,w}}(T_w/T_b)^{-0\cdot6}$ correlates the data to $\pm$ 20 per cent, the majority of the points falling within 10 per cent of the Karman–Nikuradse line when treated in this manner. This result is consistent with the fact that the right-hand side of equation (6) may be approximated by the temperature ratio raised to the $-0\cdot6$ power.

The correlation of equation 6 was tried on the bulk Reynolds number and is seen in Fig. 6. The scatter here is large but a trend is evident in that an increasing exponent is needed on the temperature ratio to correlate the data. For a

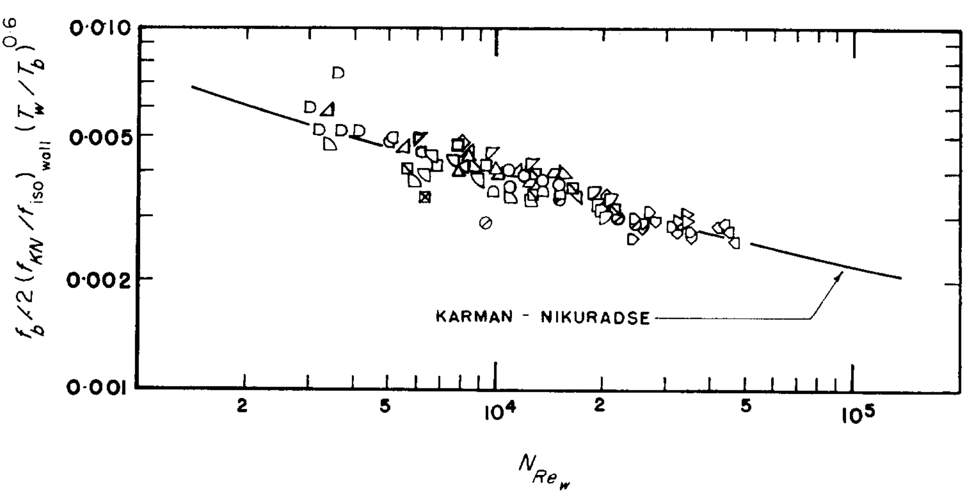

FIG. 5(a). Local friction coefficients correlated on the Karman–Nikuradse line, $16 \leqslant x/D \leqslant 114$.

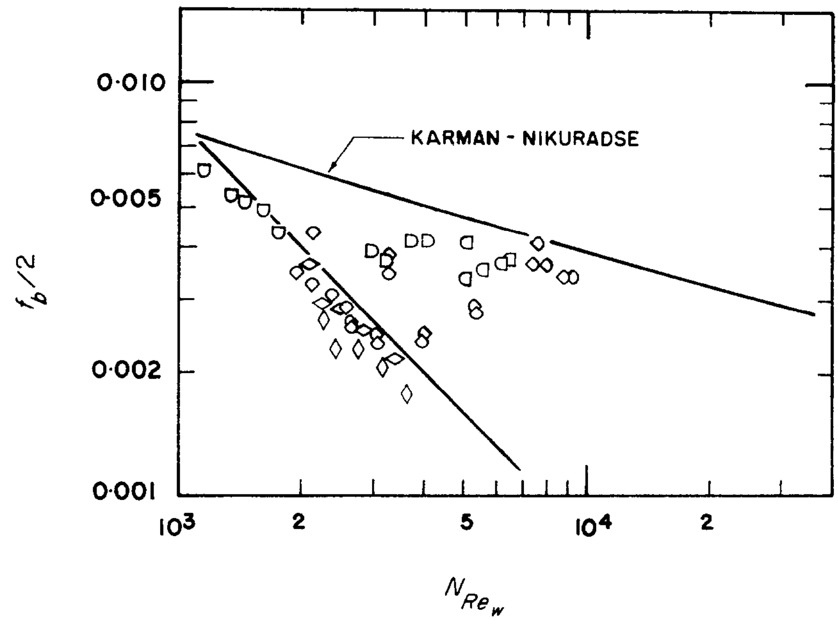

FIG. 5(b). Low $N_{Re,w}$ friction results.

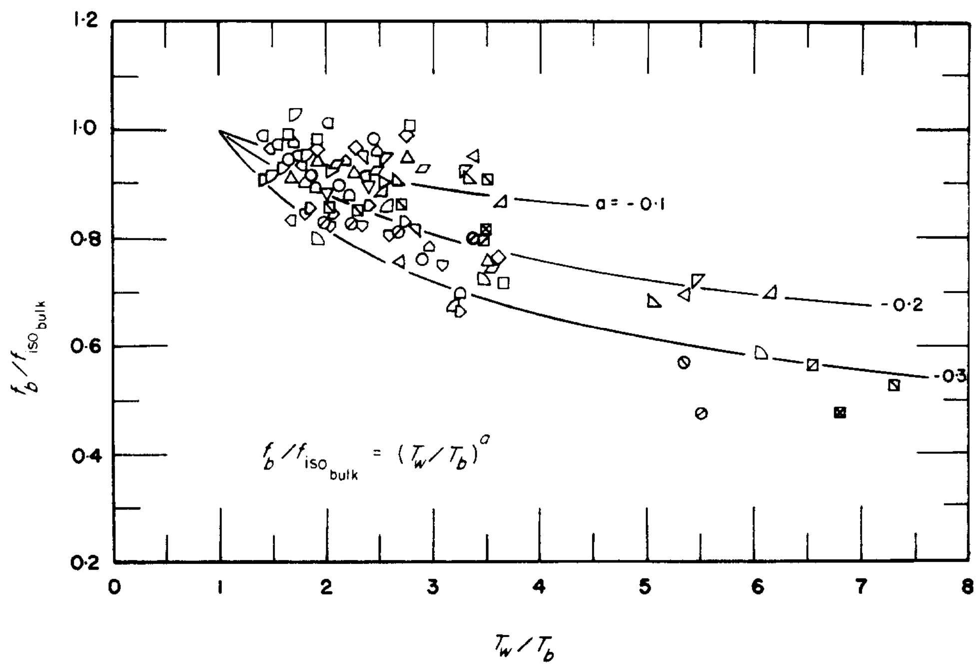

FIG. 6. Local friction coefficients normalized on isothermal bulk conditions, $16 \leqslant x/D \leqslant 114$.

ratio from about 1 to 2 the exponent would be $-0{\cdot}1$, from 2 to 4 it would be approximately $-0{\cdot}2$, and above 4 the exponent appears to be $-0{\cdot}3$. Figure 7 shows the entry correction which was made to the bulk data. This correction was determined in the same manner as the correction to the data correlated on the wall Reynolds number except that the bulk Reynolds number was used to determine the isothermal friction coefficients. The correction suggested by Magee [15] on the basis of data reaching a wall to bulk temperature ratio of 2·5 is also included. The agreement is seen to be acceptable. McEligot *et al.* [1] suggest that $f_b/f_{iso,\,b}$ should be proportional to $(T_w/T_b)^{-0{\cdot}1}$ for ratios between 1 and 2·4. This agrees with the present data as shown on Fig. 6. Lel'chuk and Dyadyakin [9] suggest that the exponent on the temperature ratio should be $-0{\cdot}16$, again for ratios up to 2·4. Their data was taken, however, with a simultaneous thermal and hydraulic entry length.

The only data which have been correlated on the wall Reynolds number are those of Dalle Donne and Bowditch [7]. They found that for temperature ratios to 2·5 the data correlated on the Karman–Nikuradse line using the modified wall Reynolds number directly without any further temperature ratio correction. The scatter was some ±25 per cent. Thus there is some disagreement between the present results and their results since our data require a temperature ratio to the $-0{\cdot}6$ power to correlate on the Karman–Nikuradse line using the wall Reynolds number.

For water, Allen and Eckert [16] and Maurer and LeTourneau [17] both found that with significant variable property effects a correlation for the friction coefficients could best be achieved on the wall Reynolds number.

Several runs were taken in which the modified wall Reynolds numbers reached values below 5000. These runs exhibited a behavior characteristic of a transition to laminar flow. Figure 5(b) includes these friction coefficient results as well as the high Reynolds number results. Our results show the laminar like runs to have friction coefficients some 10 per cent below the isothermal laminar correlation. Dalle Donne and Bowditch [7] found for laminar data that the correlation $f/2 = 8{\cdot}5/N_{Re,\,w}$ worked well. This is some 20 per cent above our results.

Some confusion remains on the use of the relation proposed by Kutateladze and Leont'ev. In reference 14 good agreement is shown for

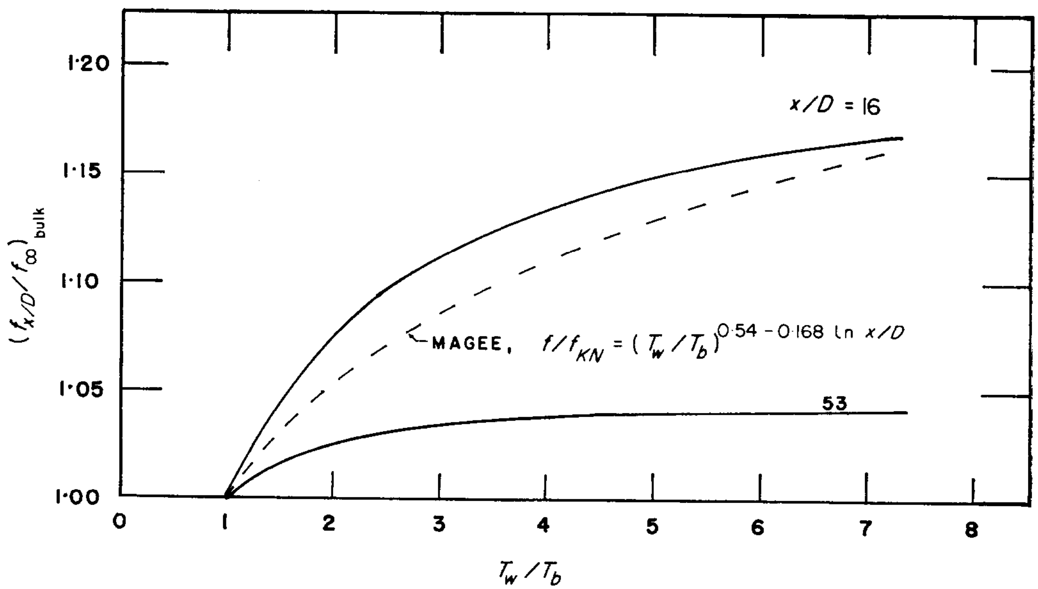

FIG. 7. Entry correction for friction coefficients based on bulk properties. Correction of Magee [15] included.

experimental results for wall to bulk temperature ratios to 2·4 but evidently with the isothermal friction coefficients based on the bulk Reynolds number. As shown on Fig. 6 our results when based on the bulk Reynolds number lie well above the correlation and the −0·6 line which approximates the correlation. It was thought that this might be caused by the inability of the limiting Reynolds number correlation to handle the finite bulk Reynolds results from the present experiment. To check this, results were also calculated using the finite, non-limiting, correlation suggested in reference 14. However, while the results fell closer to the −0·6 line they were still well above it.

As a further check the data of McEligot *et al.* [1] for air and helium and of Lel'chuk and Dyadyakin [9] for air were correlated in the present manner on the modified wall Reynolds number. Figure 8 shows the results of this reworking of their data. To determine the modified wall Reynolds number for reference 9 the bulk Reynolds numbers were multiplied by $(T_b/T_w)^{1\cdot75}$, which assumes a viscosity power law of $(T/T_{ref})^{0\cdot75}$. The results were also correlated using the entry correction determined from the

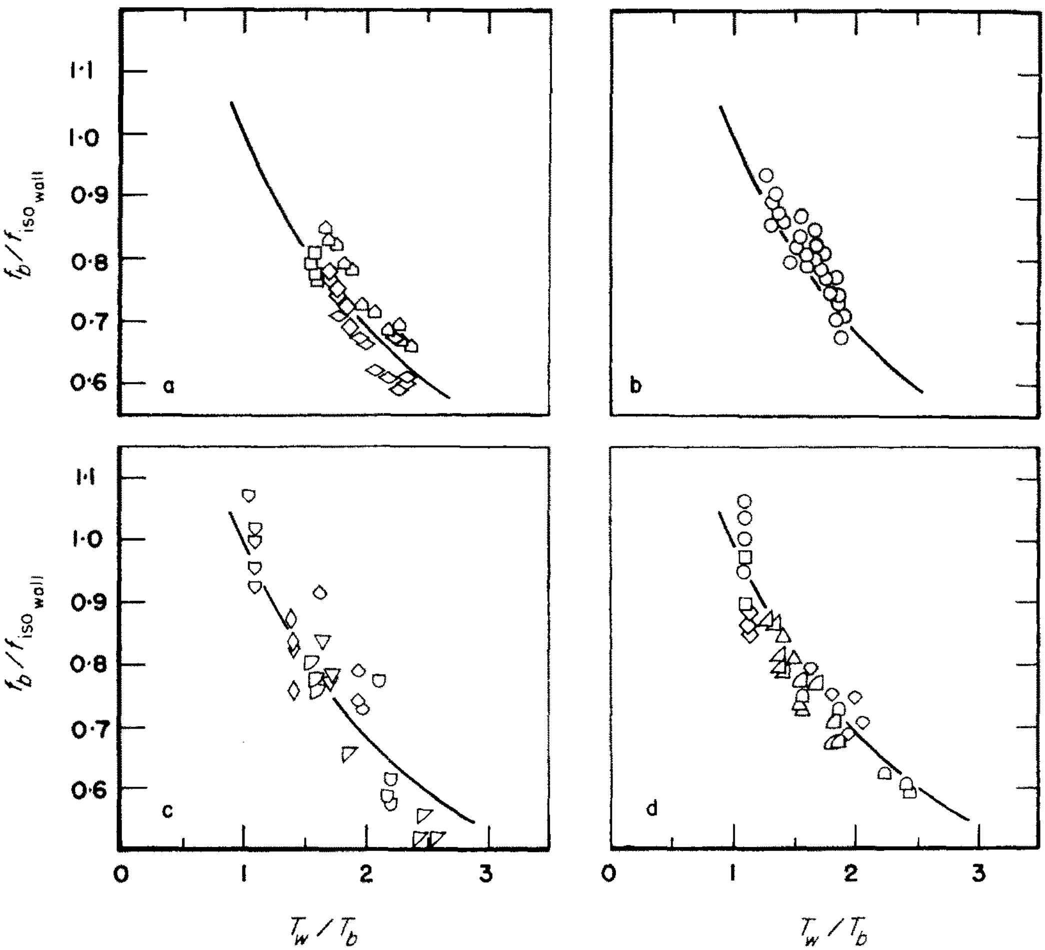

FIG. 8. Comparison of the friction results of McEligot *et al.* [1] and Lel'chuk and Dyadyakin [9] with present correlation, wall properties used throughout. (a) reference 9, entry corrected; (b) reference 9, downstream; (c) reference 1, entry corrected test section 14; (d) reference 1, downstream test section 6.

present experiment. Good agreement is seen with our data using this correlation.

Film properties, evaluated half way between the wall and the bulk temperature, were also tried. They gave results which lay between the correlation, i.e. the −0·6 line, and the bulk data. No particular correlation was evident using the film properties.

The recommended correlation for friction coefficients to wall to bulk temperature ratios of 7·5 is given by:

$$f/f_{\mathrm{iso},\,w} = \left[\frac{2}{(T_w/T_b)^{1/2}+1}\right]^2 \quad (7)$$

In simpler form this reduces to approximately:

$$f/f_{\mathrm{iso},\,w} = [T_w/T_b]^{-0\cdot 6} \quad (8)$$

The required entry correction is given in Fig. 4.

Heat-transfer results

As mentioned previously the choice of a correlation when significant variable property effects are present is one of convenience. For design purposes a bulk correlation is easier to use so the local heat-transfer results were first correlated on the bulk properties. Figure 9 shows the results for the downstream, local heat-transfer data taken from thermocouples at x/D's of 66, 89, 119, 144. The best fit for these results comes with an exponent on the wall to bulk temperature ratio of −0·7 which is slightly larger than what would correspond to the Russian prediction which Kutateladze and Leon'tev suggest should be applicable to heat transfer as well as friction coefficient results. The resulting correlation is given by:

$$N_{Nu} = 0\cdot 024\, N_{Re}^{0\cdot 8}\, N_{Pr}^{0\cdot 4}\, (T_w/T_b)^{-0\cdot 7} \quad (9)$$

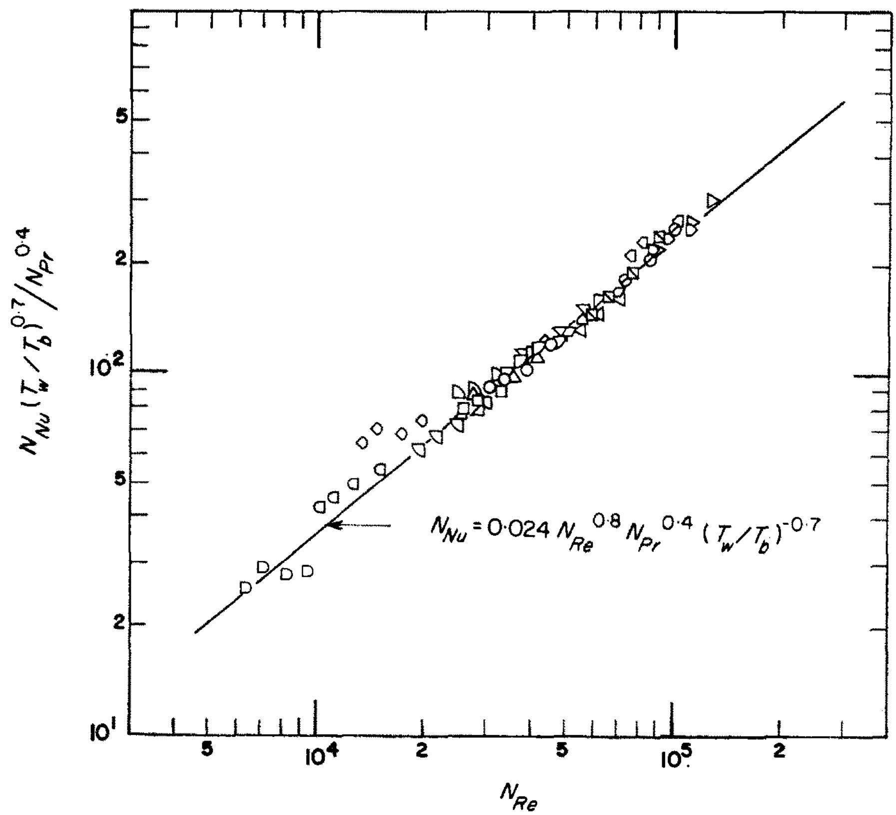

FIG. 9. Local downstream heat-transfer results based on bulk properties, $x/D > 65$.

The accuracy of the exponent on the temperature ratio is estimated to be $\pm$ 0·1. Figure 9 shows the correlation to be adequate to within $\pm$ 10 per cent except for a few runs which will be seen to have been affected by transition phenomena at the lower Reynolds numbers. The constant of 0·024 is about 5 per cent higher than the usual isothermal constant of 0·023 [18].

With this correlation for the downstream heat-transfer results as a starting point it was desired to find some suitable form of an entry parameter which would enable a correlation of all the data from the first thermocouple at an x/D of 1·2 to the end of the test section at an x/D of 144. The form of this entry parameter ideally should reduce to one which had proved satisfactory before for low wall to bulk temperature ratios. A correction of the form $[1 + (x/D)^{-0.7}]$ has been found satisfactory by both the investigators of references 1 and 19. Consequently an equation of the form $1 + (x/D)^{-0.7}\,(T_w/T_b)^a$ was tried. Figure 10 shows the final results where the data are seen to be correlated to within $\pm$20 per cent by the relation:

$$N_{Nu} = 0.024\, N_{Re}^{0.8}\, N_{Pr}^{0.4}\, (T_w/T_b)^{-0.7} \left[1 + \left(\frac{x}{D}\right)^{-0.7} \left(\frac{T_w}{T_b}\right)^{0.7}\right] \quad (10)$$

The entry correction was applied through and including an x/D of 40. Again several runs at the lower Reynolds numbers were not well correlated. These will be discussed later in this paper.

The entry correction, while adequate, does show some trends. In general the data at wall to bulk ratios from 2 to 4 are undercorrected while at the larger ratios of 6 to 7·5 the data are slightly overcorrected. A tighter correlation might be achieved by using a polynominal in

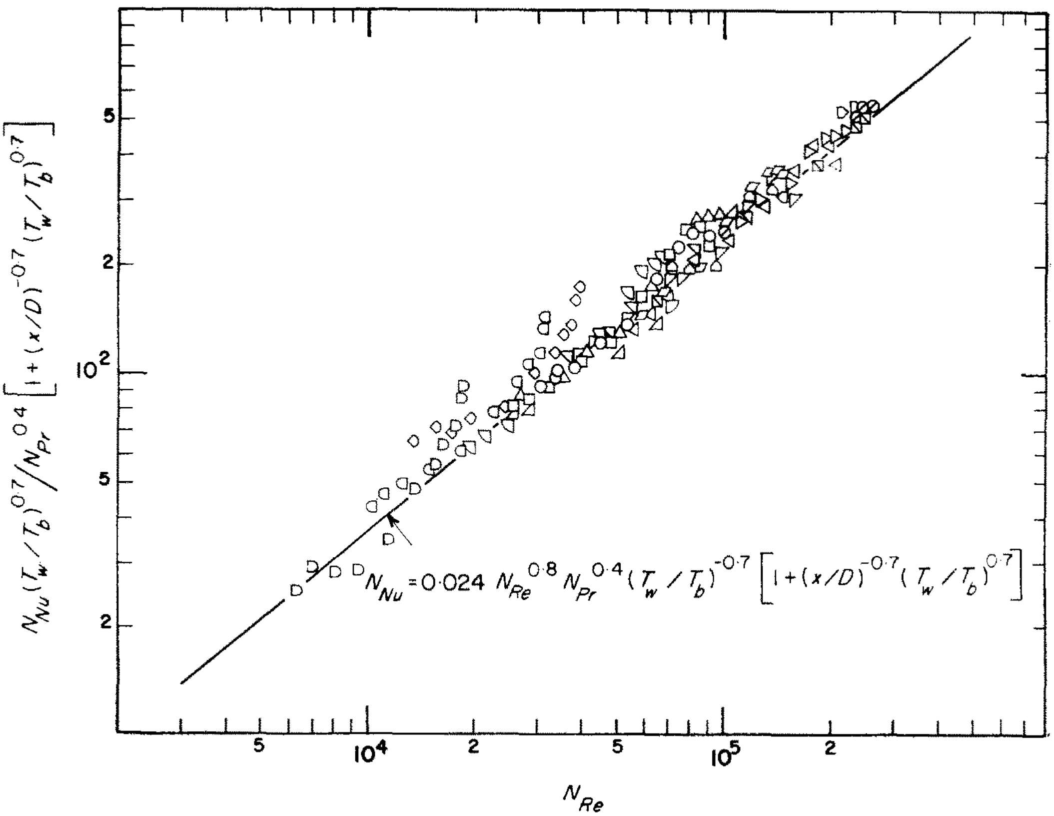

FIG. 10. Local heat-transfer results based on bulk properties, $1.2 \leqslant x/D \leqslant 144$.

T_w/T_b as a multiplier to the x/D term in the entry parameter. The convenience and simplicity of the present form, however, would seem to warrant accepting the 20 per cent scatter in the data.

A comparison of the results of references 1 and 9, both for air, for local heat transfer using bulk properties is shown in Fig. 11. In this Figure the downstream data of McEligot *et al.* and of Lel'chuk and Dyadyakin are correlated in the manner of the present experiment. An insert on the Figure shows the improvement which is found by adding a wall to bulk temperature factor to the entry parameter. The agreement between the three experiments is seen to be good; McEligot's data lies some 5 per cent below the present data in general as does the Russian data.

Another form of a correlation for local heat-transfer results has been found by Dalle Donne and Bowditch [6, 7] in which they also use bulk properties with a wall to bulk temperature ratio. However, the exponent on the temperature ratio in their correlation is of the form $a + b\,(x/D)$ so that any entry effect is included in the exponent. A similar form was tried with the present data; however, the exponent ceased to be a function of test-section length after some 40 diameters so that it was felt that an entry parameter with x/D included was more representative of the physical situation. As pointed out in reference 6 the average exponent on the temperature ratio for their tube is about $-0{\cdot}8$ in comparison to our exponent of $-0{\cdot}7$. The data of reference 6 includes wall to bulk ratios to 2·4. The correlating constant for their local heat-transfer data is the same as ours, 0·024

It thus appears that good agreement is found

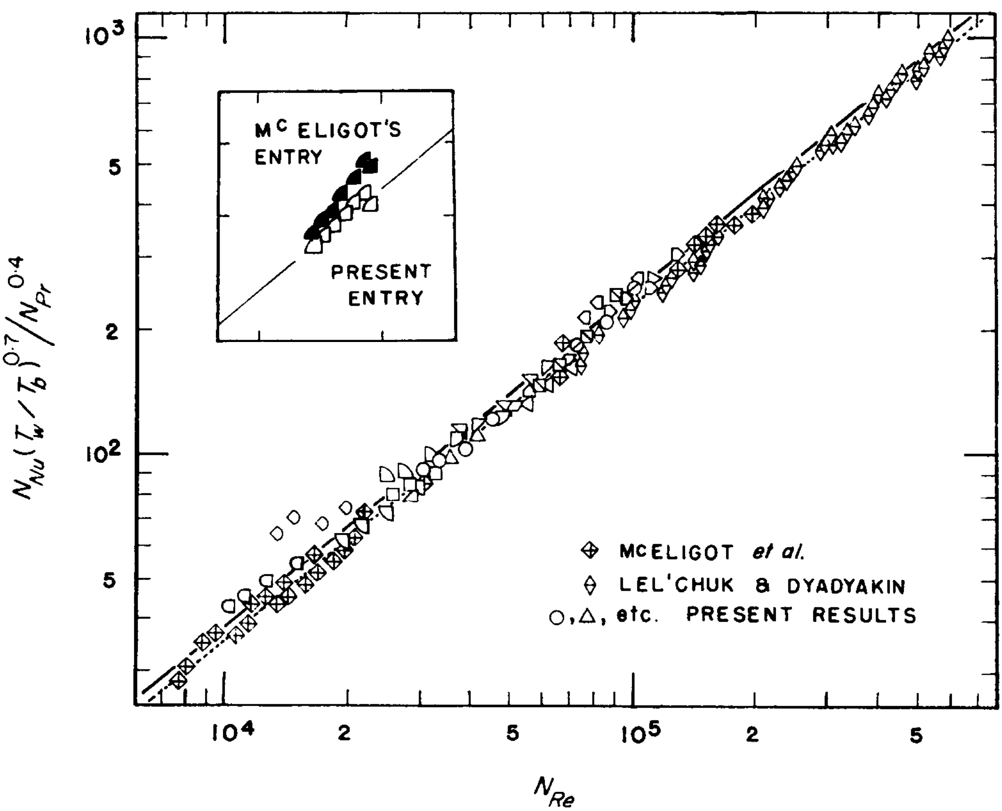

FIG. 11. Comparison of the results of McEligot *et al.* [1] and Lel'chuk and Dyadyakin [9] with present experiment. Bulk properties used throughout, downstream data only. Solid line is correlation constant of 0·024, dotted line 0·023. Insert shows improvement of entry correction using T_w/T_b in entry parameter.

between the present results, correlated with bulk properties, and the results of references 1, 6, 7 and 9.

The results of McCarthy and Wolf [4] for incremental average heat-transfer data correlate on a wall to bulk temperature ratio to the $-0{\cdot}55$ power for temperature ratios to 9. A correlation of the present data with this exponent was also tried. The downstream data could be adequately correlated to within ± 15 per cent with this power and a constant of 0·022. However, a systematic deviation occurred in the data when the entry correction was applied. A correlation of the form of equation (10), but with a value of 0·55 for the exponent on the wall to bulk temperature ratio in both places in which the ratio appears, proved to work well except that points with wall to bulk temperature ratios of 5 or larger fell some 25 per cent below the correlation. When the value of 0·7 was used for the exponent in both the entry and downstream correlation no such systematic deviation was noted. One might argue that the high wall to bulk points occur at the peak in the wall temperature profile, see Fig. 1, where there is dissimilarity between the heat loss runs and the flow runs. Since, with the high Reynolds number runs, the total heat loss was less than 10 per cent this dissimilarity should be a second order effect. The recommended correlation using bulk properties is thus equation (10).

Another correlation for heat-transfer data which has had common acceptance uses the modified wall Reynolds number. This method was found to be satisfactory with the present results and is shown in Fig. 12. In this case all of the properties are evaluated at the wall temperature in both the Reynolds numbers and the heat-transfer parameters. A comparison with the results of other authors may be made from

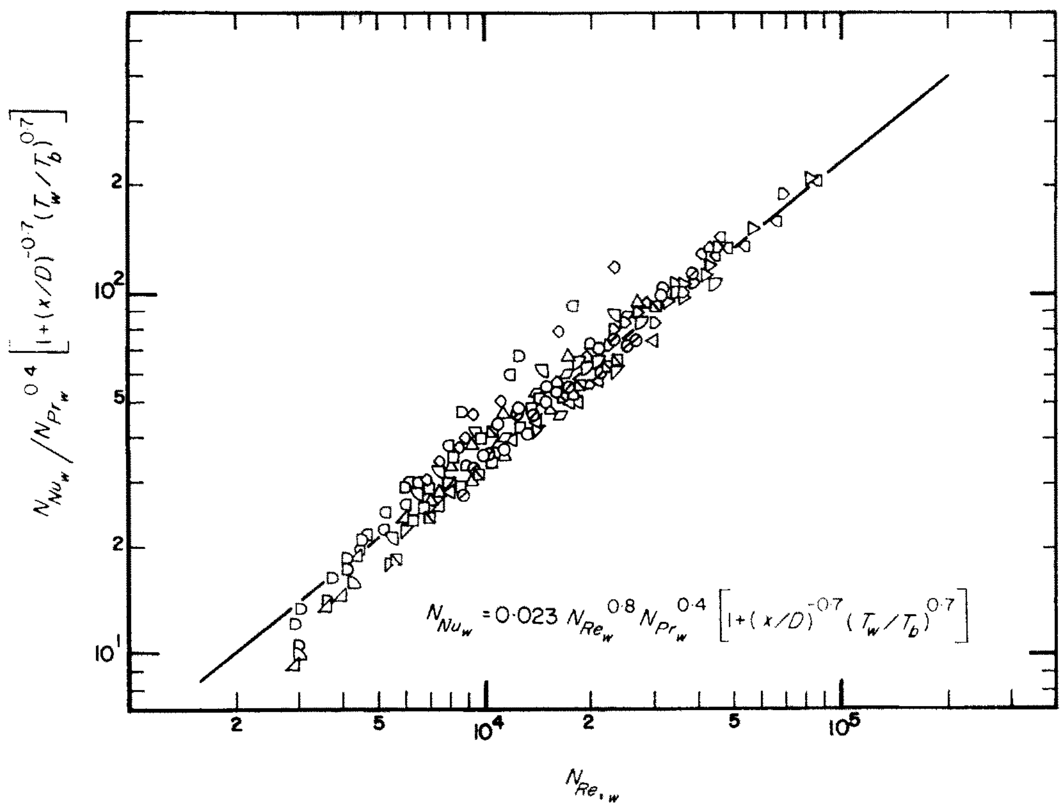

FIG. 12. Local heat-transfer results based on wall properties, $1{\cdot}2 \leqslant x/D \leqslant 144$.

Table 1. Previously data have been correlated in this manner with a constant that ranges from 0·0265 for the data of Kirchgessner and Taylor [8, 19] to 0·0208 for that of Dalle Donne and Bowditch [7]. The present data are seen to be correlated by the relation:

$$N_{Nu,\,w} = 0{\cdot}023\, N_{Re,\,w}^{0\cdot8}\, N_{Pr,\,w}^{0\cdot4} \left[1 + \left(\frac{x}{D}\right)^{-0\cdot7} \left(\frac{T_w}{T_b}\right)^{0\cdot7}\right] \quad (11)$$

This includes data with wall to bulk ratios of 7·5 in the tube as a whole and 3 in the downstream region. For the wall correlation the entry correction was applied through and including an x/D of 24. A plot of the local heat-transfer results versus x/D showed that the entry effect was not felt as far downstream using wall properties as using bulk properties, thus the difference in applying the entrance correction parameter. The downstream results using wall properties correlated best with a constant of 0·022 in equation (11).

A film correlation was also tried but the scatter was markedly larger than that found with the wall correlation. Similar behavior has also been found by Taylor [8] and Wieland [5].

The fact that satisfactory correlations can be achieved with both bulk and wall properties should not be surprising since one can start with either correlation and attain the second one by using power laws on temperature for the viscosity and conductivity and the perfect gas law for the density. Using the power laws corresponding to the temperatures in this experiment, one can deduce an exponent of about $-0{\cdot}7$ on the wall to bulk temperature ratio from the wall correlation, equation (11).

Entry region

The entry parameter used in correlating the data from the present experiment has a very large effect at small x/D and large wall to bulk temperature ratios. For example, at an x/D of 4·6 the correction amounts to a factor of 2 at a temperature ratio of 4. In order to check this entry correction the data of both references 1 and 9 were correlated in this manner. In both cases the correlation was good. In addition a comparison was made with the results of references 5, 6 and 7 in so far as it was possible to do so. In reference 7, Dalle Donne and Bowditch suggest that for the correlation with the modified wall Reynolds number an entry parameter with the form and magnitude of $(1 + 4D/x)$ should be used, in reference 6 they suggest $(1 + 6{\cdot}2\ D/x)$. These give a correction independent of the temperature ratio but of the same size as the correction made in the present experiment. Wieland [5] shows a plot of the factor $N_{Nu,w}/N_{Re,w}^{0\cdot8}\ N_{Pr,w}^{0\cdot4}$ versus x/D for two runs with a low and high wall to bulk temperature ratio. Our data agree closely with these results at a corresponding temperature ratio; and consequently, it is thought that our entry correction will work with his data.

Just why the entry effect is so accented by larger wall to bulk ratios is not clear. There is good agreement for constant property results with the entry correlation of Sparrow *et al.* [20] which is much smaller than the parameter found here. It would appear that future work is warranted in the area of entry effects at high wall to bulk temperature ratios.

Average results

Figure 13(a) and (b) show the average heat-transfer and friction coefficient results calculated in the manner of Humble *et al.* [3]. In both cases the downstream local correlation works for the average results. It should be noted that the average wall to bulk temperature ratio used in these results is the average wall temperature divided by the average bulk temperature, not the average of the wall to bulk temperature ratio. No entry correction is needed for these results which are based on an L/D of 144 for the heat transfer and a length of test section from an x/D of 16 to 114 for the friction coefficients.

Transition behavior

Because of the large increase of viscosity due to the rise in temperature, the local bulk Reynolds number experienced a large decrease through the tube length. Although the bulk Reynolds number never came near the critical value, it was apparently possible to obtain a

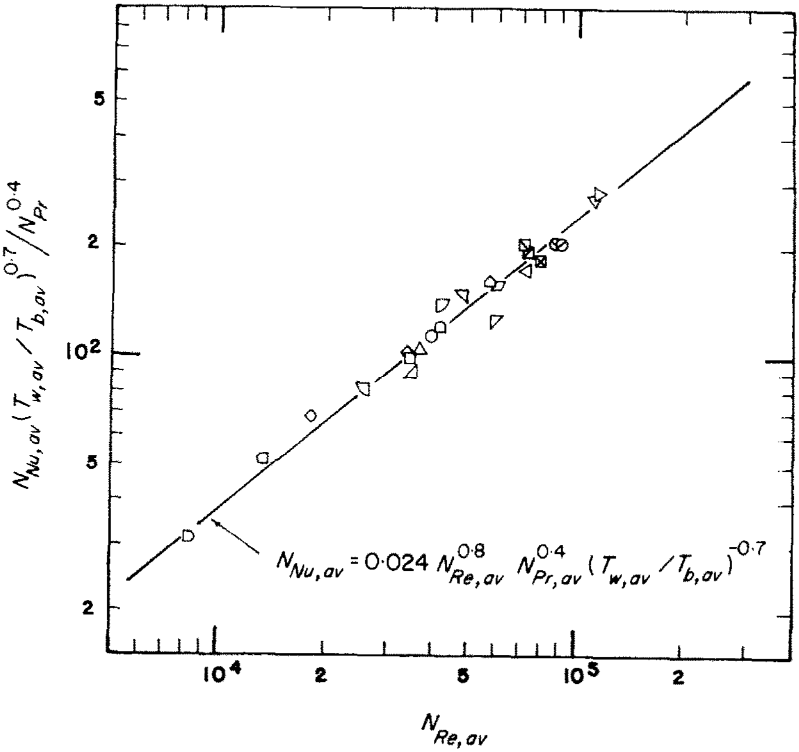

FIG. 13(a). Average heat-transfer results for T_w/T_b to 2·46.

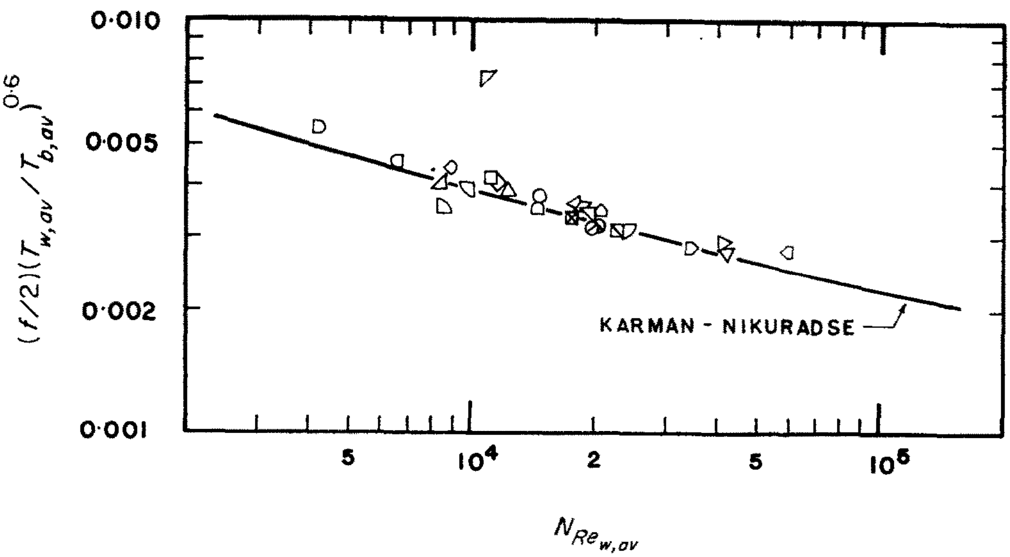

FIG. 13(b). Average friction coefficient results for T_w/T_b to 2·46.

Table 3. Summary of data

Symbol	Run	Bulk inlet N_{Re}	Bulk exit N_{Re}	Minimum wall N_{Re}	$T_{w_{max}}$ (°F)	$\frac{T_{wall}}{T_{bulk_{max}}}$	Comments
○	102	98 000	33 600		795	2·95	
□	103	97 000	28 000		1160	3·73	
◇	104	97 000	28 000		1158	3·69	
△	105	103 000	30 000		1000	3·60	
▱	106	160 000	52 000		1000	3·63	
▷	107	236 000	100 000		448	2·58	
▽	108	223 000	99 000		451	2·44	
◁	109	226 000	62 000		1385	5·53	
◺	110	211 000	62 000		1396	5·20	
◸	111	170 000	44 000		1376	5·61	
◹	112	120 000	42 000		585	2·88	
◿	113	122 000	29 000		1847	6·48	
◺	114	122 000	27 000		1856	6·24	
◸	115	76 000	37 000		157	1·84	
◹	116	72 700	21 000		867	3·37	
◿	117	72 000	21 000		875	3·34	
⌓	118	111 000	35 000		861	3·31	
D	119	18 700	6400	2900	444	2·29	
⊂	120	18 300	4300	1020	1560	3·78	runs 119 to
⊃	121	33 000	11 000	4500	580	2·50	127 show some
▽	122	33 000	9000	1730	1622	4·04	transition
◁	123	43 000	15 000	6800	419	2·28	effects.
△	124	44 000	12 000	2530	1141	3·77	
▷	125	44 000	12 000	2530	1124	3·76	
◊	126	43 000	10 000	1890	1588	4·36	
◇	127	45 000	11 000	1750	1571	4·78	
⌂	128	142 000	48 000		558	3·00	
▷	129	268 000	96 000		560	3·23	
▽	130	272 000	97 000		560	3·19	
◁	131	157 000	83 000		70	1·80	
—	132	none	none		none	none	runs 133 to
—	133	352 000	198 000		25·0	1·76	135 show
—	134	357 000	203 000		4·0	1·78	condensation
—	135	359 000	204 000		10·5	1·78	effects
⌀	136	279 000	74 800		1295	5·74	
⍉	137	271 000	74 800		1299	5·55	
⊠	138	266 000	67 000		1658	7·08	
⊠	139	266 000	67 000		1648	6·85	
⊠	140	266 000	67 000		1717	7·53	

turbulent to laminar transition in the test section when running at moderate inlet Reynolds numbers and high wall to bulk temperature ratios. The effect of such a transition on the friction coefficients can be seen in Fig. 5(b) where the local coefficients are plotted versus the modified wall Reynolds number. With this particular choice of a Reynolds number the typical transition region is seen in the usual range of Reynolds number. Table 3, runs 119 to 127, shows the range of variables for those runs which show some effects of either laminar flow or transitional flow. For runs 119, 121, and 123 the friction coefficients seem to be only moderately affected. However, these runs show considerable effects on the local heat-transfer results as seen in Figs. 9, 10, and 12. The other runs, between 119 and 127, are not shown on the heat-transfer plots as they are clearly in the laminar or transitional regime.

CONCLUSIONS

Results for both local and average friction and heat-transfer coefficients have been correlated for local wall to bulk temperature ratios to 7·5. The recommended correlations are:

Friction coefficients

$$f/f_{iso,\,w} = \left[\frac{2}{(T_w/T_b)^{1/2} + 1}\right]^2$$

See Fig. 4 for the entry parameter for $x/D < 55$.

Heat transfer

Bulk correlation

$$N_{Nu} = 0{\cdot}024\, N_{Re}^{0\cdot8}\, N_{Pr}^{0\cdot4}\, (T_w/T_b)^{-0\cdot7} \left[1 + \left(\frac{x}{D}\right)^{-0\cdot7} \left(\frac{T_w}{T_b}\right)^{0\cdot7}\right]$$

with the same correlation without the entry parameter for $x/D > 40$.

Wall correlation

$$N_{Nu,\,w} = 0{\cdot}023\, N_{Re,\,w}^{0\cdot8}\, N_{Pr,\,w}^{0\cdot4} \left[1 + \left(\frac{x}{D}\right)^{-0\cdot7} \left(\frac{T_w}{T_b}\right)^{0\cdot7}\right]$$

with the same correlation without the entry parameter for $x/D > 24$.

ACKNOWLEDGEMENTS

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REFERENCES

1. D. M. McEligot, P. M. Magee and G. Leppert, Effect of large temperature gradients on convective heat transfer: the downstream region, *ASME* paper 64-*HT*-12.
2. M. E. Davenport and G. Leppert, The effect of transverse temperature gradients on the heat-transfer and friction for laminar flow of gases, *ASME* paper 64-*HT*-10.
3. L. V. Humble, W. H. Lowdermilk and L. G. Desmon, Measurements of average heat transfer coefficients for subsonic flow of air in smooth tubes at high surface and fluid temperatures, *NACA* report 1020 (1951).
4. J. R. McCarthy and H. Wolf, The heat transfer characteristics of gaseous hydrogen and helium, *RR*-60-12, Rocketdyne, Canoga Park, California (1960).
5. W. F. Wieland, Measurement of local heat transfer coefficients for flow of hydrogen and helium in a smooth tube at high surface to fluid bulk temperature ratios, *AIChE Symposium on Nuclear Engineering Heat Transfer*, Chicago (1962).
6. M. Dalle Donne and F. H. Bowditch, High temperature heat transfer, *Nucl. Engng* **8**, 20–29 (1963).
7. M. Dalle Donne and F. H. Bowditch, Experimental local heat transfer and friction coefficients for sub-sonic laminar, translational, and turbulent flow of air or helium in a tube at high temperatures, Dragon Project D.P. 184 (1963).
8. M. F. Taylor, Local heat transfer measurements for forced convection of hydrogen and helium at surface temperatures up to 5600°R, *Heat Transfer and Fluid Mechanics Institute*, p. 251, Stanford University Press, Stanford, California (1963).
9. V. L. Lel'chuk, and B. V. Dyadyakin, Heat-transfer from a wall to a turbulent current of air, inside a tube and the hydraulic resistance at large temperature differentials, from *Problems in Heat Transfer*, *AEC TR*-4511 (1962).
10. D. M. McEligot, Effect of large temperature gradients on turbulent flow of gases in the downstream region of tubes, Ph.D. Thesis, Mechanical Engineering Department, Stanford University, Stanford, California (1963).
11. M. E. Davenport, P. M. Magee and G. Leppert, Thermocouple attachment to a direct current heater, *J. Heat Transfer* **84**, 187 (1962).
12. J. C. Hilsenrath, W. Beckett, W. S. Benedict, L. Fano, H. J. Hoge, J. F. Masi, R. L. Nuttall, Y. S. Touloukian and H. W. Wooley, *NBS Circular 564* (1955).
13. A. J. W. Smith, On the correlation and presentation of information on turbulent variable property heat-transfer in tubes, *J. Roy. Aeronaut. Soc.* **66**, 397–400 (1962).
14. S. S. Kutateladze and A. I. Leont'ev, *Turbulent Boundary Layers in Compressible Gases* (Trans. D. B. Spalding). Arnold, London (1964).
15. P. M. Magee, Turbulent heat transfer and friction in the thermal entrance region of a tube with large transverse temperature gradients, Ph.D. Thesis, Mechanical Engineering Department, Stanford University (1964).
16. R. W. Allen and E. R. G. Eckert, Friction and heat transfer measurements to turbulent pipe flow of water (*Pr* 7 and 8) at uniform wall heat flux, *J. Heat Transfer* **86**, 301–310 (1964).
17. G. W. Maurer and B. LeTourneau, Friction factors for fully developed turbulent flow in ducts

with and without heat transfer, *J. Basic Engng* **86**, 627–636 (1964).
18. W. H. McAdams, *Heat Transmission*, 3rd ed. McGraw-Hill, New York (1954).
19. M. F. Taylor and T. A. Kirchgessner, Measurements of heat transfer and friction coefficients for helium flowing in a tube at surface temperatures up to 5900°R, NASA *TN D*-133 (1959).
20. E. M. Sparrow, T. M. Hallman and R. Siegel, Turbulent heat transfer in the thermal entrance region of a pipe with uniform heat flux, *Appl. Sci. Res.* 7A, 37–52 (1957).

Résumé—On rapporte les mesures de frottement local et de transport de chaleur avec un flux de chaleur constant dans un tube circulaire chauffé électriquement. De l'azote refroidi préalablement en écoulement turbulent a été employé pour obtenir des rapports de la température locale de paroi à la température globale allant de 1,24 à 7,54. La (section) d'essai en Inconel avait une région d'entrée hydrcdynamique de 40 diamètres de long. Les nombres de Reynolds globaux d'entrée couvraient la gamme de 18 300 à 279000. Des corrélations en fonction à la fois des nombres de Reynolds globaux et des nombres de Reynolds mcdifiés rapportés à la paroi sont présentées pour les coefficients de frottement locaux; les résultats du transport de chaleur local sont présentés en fonction du rapport de la température de la paroi à la température globale avec des propriétés globales et aussi directement sur le nombre de Reynolds modifié rapporté à la paroi. On montre que les résultats du frottement et du transport de chaleur sont tous les deux affectés d'une façon sensible par l'influence des propriétés variables. Les effets de la région d'entrée sont discutés à la fois pour les oecfficients de transport de chaleur et de frottement.

Zusammenfassung—Für lokale Reibungsbeiwerte und Wärmeübergangszahlen bei konstanter Wärmestromdichte in einem elektrisch beheizten Kreisrohr werden Messungen mitgeteilt. Um ein örtliches Temperaturverhältnis zwischen Wand und angrenzendem Medium vcn 1,24 bis 7,54 zu erhalten, wurde vorgekühlter turbulent strömender Stickstoff verwendet. Die Inconel-Versuchsstrecke hatte eine hydrcdynamische Einlauflänge von 40 Durchmessern und eine beheizte Länge von 160 Durchmessern. Die Reynoldzsahlen des Mediums am Eintritt überdeckten einen Bereich von 18 300 bis 279 000. Für die lokalen Reibungsbeiwerte werden Beziehungen in Termen sowohl mit den Reynoldszahlen des Mediums wie auch mit den mcdifizierten Wand-Reynoldszahlen angegeben. Die Ergebnisse des lokalen Wärmeübergangs werden in Termen des Temperaturverhältnisses zwischen Wand und angrenzendem Medium mit den Stoffwerten des Mediums und auch direkt auf die mcdifizierte Wand-Reynoldszahl bezcgen aufgeführt. Die Ergebnisse sowohl für die Reibung als auch für den Wärmeübergang zeigen den deutlichen Einfluss der variablen Stoffwerte. Die Wirkung des Einlaufbereichs sowohl auf die Wärmeübergangszahl als auch auf den Reibungsbeiwert wird diskutiert.

Аннотация—Приведены измерения локальных коэффициентов трения и теплообмена при постоянном тепловом потоке в электронагреваемой круглой трубе. Предварительно охлажденный азот в турбулентном потоке использовался для получения отношений локальных температур стенки и объема в диапазоне 1,24–7,54. Размеры участков стабилизации стенда, изготовленного из инконеля, следующие: гидродинамический—40 диаметров, тепловой-160 диаметров. Объемные числа Рейнольдса на входе охватывали диапазон от 18300 до 279000. Представлены связи, исходя из объемных чисел Рейнольдса, а также модифицированных чисел Рейнольдса на стенке, для локальных коэффициентов трения; коэффициенты локального теплообмена представлены в виде функции температур на стенке к объемной температуре, учитывая свойства жидкости в объеме, отнесенной непосредственно к модифицированному числу Рейнольдса. Показано, что переменные свойства значительно влияют на результаты трения, а также теплообмена. Обсуждается влияние входного участка на коэффициенты теплообмена и трения.